بومنزه و ثنان

السلسلة3)

التمرين16) 1) $|a|=2$ ومنه فان $a=2(0+1i)$ نضع $\cos\theta=0$ و $Sin\theta=1$

ومنه $\theta\equiv\frac{\pi}{2}[2\pi]$ و $a=\left[2,\frac{\pi}{2}\right]$ بالتالي

$|b|=2$ ومنه فان $b=2\left(\frac{\sqrt{3}}{2}+\frac{1}{2}\right)=2\left(\sin\frac{\pi}{6}+\cos\frac{\pi}{6}\right)=\left[2,\frac{\pi}{6}\right]$

$|c|=2$ ومنه فان

$$c=2\left(\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}\right)=2\left(\cos\frac{\pi}{4}+\sin\frac{\pi}{4}\right)$$
$$=\left[2,\frac{\pi}{4}\right]$$

(2

$$(\alpha)\quad \frac{c}{b}=\frac{\sqrt{2}(1+i)}{\sqrt{3}+i}=\frac{\sqrt{2}(1+i)(\sqrt{3}-i)}{4}$$
$$=\frac{\sqrt{6}+\sqrt{2}}{4}+i\,\frac{\sqrt{6}-\sqrt{2}}{4}$$

ولدينا

$$(\beta)\quad \frac{c}{b}=\frac{\left[2,\frac{\pi}{4}\right]}{\left[2,\frac{\pi}{6}\right]}=\left[1,\frac{\pi}{12}\right]=\cos\left(\frac{\pi}{12}\right)+\sin\left(\frac{\pi}{12}\right)$$

بعد مطابقة (α) و (β) نحصل على

$$\cos\frac{\pi}{12}=\frac{\sqrt{6}+\sqrt{2}}{4}$$
$$\sin\frac{\pi}{12}=\frac{\sqrt{6}-\sqrt{2}}{4}$$

(3 $\widehat{\left(\overrightarrow{OB},\overrightarrow{OC}\right)}\equiv\arg\left(\frac{c}{b}\right)[2\pi]\equiv\arg\left[1,\frac{\pi}{12}\right][2\pi]\equiv\frac{\pi}{12}[2\pi]$

التمرين17)

(1

$$z=2\cos^2\theta+i\sin 2\theta=2\cos^2\theta+i\,2\sin\theta\cos\theta$$
$$=2\cos\theta(\cos\theta+i\sin\theta)$$

بما أن $\theta\in\left]-\frac{\pi}{2},\frac{\pi}{2}\right[$ فان $\cos\theta\succ 0$ ومنه $|z|=2\cos\theta$ و $\arg z\equiv\theta[2\pi]$

(2

$$Z=\frac{1}{z^2}=\frac{1}{[2\cos\theta,\theta]^2}=\frac{1}{[4\cos^2\theta,2\theta]}=\left[\frac{1}{4\cos^2\theta},-2\theta\right]$$

ومنه فان $|Z|=\frac{1}{4\cos^2\theta}$ و $\arg Z\equiv-2\theta[2\pi]$

(3

$$\frac{1}{4}(1-tg^2\theta)-\frac{i}{2}tg\theta=\frac{1}{4}(\frac{\cos^2\theta-\sin^2\theta}{\cos^2\theta})-\frac{i}{2}\frac{\sin\theta}{\cos\theta}$$
$$=\frac{1}{4\cos^2\theta}(\cos(-2\theta)+i\sin(-2\theta))$$
$$=\left[\frac{1}{4\cos^2\theta},-2\theta\right]$$
$$=Z$$

4) لدينا $M(z)$ ومنه $OM=|z|=2\cos\theta$ وبالتالي فان النقطة $M(z)$ تنتمي الى الدائرة التي مركزها O وشعاعها $R=2\cos\theta$

التمرين19 1)-

$$u^2=\left(1+\sqrt{3}\right)^2-2i\left(1+\sqrt{3}\right)\left(1-\sqrt{3}\right)-\left(1-\sqrt{3}\right)^2$$
$$=4\sqrt{3}+4i=\left[8,\frac{\pi}{6}\right]$$

نضع $u=[r,\theta]$ بحيث $\cos\theta\succ 0$ و $\sin\theta\succ 0$ لأن $1+\sqrt{3}\succ 0$ و $\sqrt{3}-1\succ 0$

$$u^2=\left[r^2,2\theta\right]=\left[8,\frac{\pi}{6}\right]\Rightarrow r^2=8 \text{ و } \theta=\frac{\frac{\pi}{6}+2k\pi}{2}\ ,\ k\in\{1,2\}$$

$$\Rightarrow u=\left[2\sqrt{2},\frac{\pi}{12}\right] \text{ أو } u=\left[2\sqrt{2},-\frac{11\pi}{12}\right]$$

وبما أن $\cos\left(-\frac{11\pi}{12}\right)\prec 0$ فان $u=\left[2\sqrt{2},\frac{\pi}{6}\right]$

2) لدينا $\Delta=\bar{u}^2-4\times 2\sqrt{3}=\overline{4\sqrt{3}+4i}-8\sqrt{3}=-4\sqrt{3}-4i$
$$=-u^2=(iu)^2$$

و منه $z_0=\frac{\bar{u}+iu}{2}=1+i$, $z_1=\frac{\bar{u}-iu}{2}=\sqrt{3}-i\sqrt{3}$

وبالتالي $S=\{z_0,z_1\}$

3) لدينا $z_0=1+i=\left[\sqrt{2},\frac{\pi}{4}\right]$ و $z_1=\sqrt{3}-i\sqrt{3}=\left[\sqrt{6},-\frac{\pi}{4}\right]$

و منه $\frac{1}{z_0}+\frac{1}{z_1}=\frac{z_0+z_1}{z_0z_1}=\frac{\frac{-b}{a}}{\frac{c}{a}}=\frac{-b}{a}$ و $z_1^2+z_0^2=\left(z_0+z_1\right)^2-2z_0z_1$

$$\frac{1}{z_0}+\frac{1}{z_1}=\bar{u}\ =\left[2\sqrt{2},-\frac{\pi}{6}\right]$$

$$z_1^{\,2}+z_0^{\,2}=\left(\frac{-b}{a}\right)^2-2\frac{c}{a}=\frac{b^2-2ac}{a^2}=\frac{\bar{u}^{2}-4\sqrt{3}}{a^2}=-4i\ =\left[4,-\frac{\pi}{2}\right]$$

<u>التمرين21)</u> 1) لدينا

$$z_0z_1=\frac{c}{1}=i\Rightarrow |z_0||z_1|=|i|=1 \text{ و } \arg\left(z_0z_1\right)\equiv\frac{\pi}{2}[2\pi]$$

$$\Rightarrow |z_0||z_1|=1 \quad \text{و} \quad \arg z_0+\arg z_1\equiv\frac{\pi}{2}[2\pi]$$

2) أ- لدينا $z_0=e^{i\theta}$ وبمأن $z_0z_1=i$ فان $z_1=\frac{i}{z_0}=\frac{e^{i\frac{\pi}{2}}}{e^{i\theta}}=e^{i\left(\frac{\pi}{2}-\theta\right)}$

ب- انطلاقا من علاقة أولير فانه لدينا $\cos\alpha=\frac{e^{i\alpha}+e^{-i\alpha}}{2}$

$$z_0+z_1=a\Rightarrow a=e^{i\theta}+e^{i(\frac{\pi}{2}-\theta)}$$

$$\Rightarrow a=e^{i\frac{\pi}{4}}(e^{i(\theta-\frac{\pi}{4})}+e^{i(\frac{\pi}{2}-\theta-\frac{\pi}{4})})$$

$$\Rightarrow a=e^{i\frac{\pi}{4}}(e^{i(\theta-\frac{\pi}{4})}+e^{-i(\theta-\frac{\pi}{4})})$$

$$\Rightarrow a=e^{i\frac{\pi}{4}}\cos\left(\theta-\frac{\pi}{4}\right)$$

3) أ- نفترض أن $z_0=\frac{\sqrt{3}}{2}+\frac{i}{2}$ ومنه فان $z_0=e^{i\frac{\pi}{6}}$ وبا ستعمال السؤال2)أ-

نستنتج

أن $z_1=e^{i\left(\frac{\pi}{2}-\frac{\pi}{6}\right)}=e^{i\frac{\pi}{3}}=\left[1,\frac{\pi}{3}\right]$ (نضع $\theta=\frac{\pi}{6}$) ومنه $z_1=\frac{1}{2}+i\frac{\sqrt{3}}{2}$

ب- نضع $\theta=\frac{\pi}{6}$ ومنه $a=2\cos\left(\frac{\pi}{6}-\frac{\pi}{4}\right)e^{i\frac{\pi}{4}}=\left(\frac{\sqrt{2}}{2}+i\frac{\sqrt{2}}{2}\right)2\cos\left(\frac{\pi}{12}\right)$

$$a=z_0+z_1=\left(\sqrt{2}+i\sqrt{2}\right)\cos\frac{\pi}{12}=\frac{\sqrt{3}}{2}+i\frac{1}{2}+\frac{1}{2}+i\frac{\sqrt{3}}{2}=\left(\frac{\sqrt{3}+1}{2}\right)+i\left(\frac{\sqrt{3}+1}{2}\right)$$

ومنه $\cos\frac{\pi}{12}=\frac{1+\sqrt{3}}{2}(1+i)\times\frac{1}{\sqrt{2}(1+i)}=\frac{\sqrt{2}+\sqrt{6}}{4}$

وبالتالي

$$\sin\frac{\pi}{12}=\sqrt{1-\left(\frac{\sqrt{2}+\sqrt{6}}{4}\right)^2}=\sqrt{\frac{8-2\sqrt{12}}{16}}=\sqrt{\frac{\left(\sqrt{6}-\sqrt{2}\right)^2}{16}}=\frac{\sqrt{6}-\sqrt{2}}{4}$$